Heterogeneous Firms, Financial Frictions, and the Transmission of Foreign Shocks *

Yijia Chen[†] Ida K. Haavi[‡]

July 28, 2025

Abstract

This paper analyzes how firm-level financial frictions shape the transmission of foreign borrowing shocks in a small open economy. We embed dividend non-negativity and collateral constraints into a heterogeneous firm New Keynesian (HANK) model, calibrated to key moments of the Hungarian economy just prior to the Global Financial Crisis. We introduce a small Sudden Stop shock, an unanticipated 1% drop in foreign capital inflows followed by a gradual recovery, and examine its aggregate effects. Our results show that a high prevalence of collateral-constrained firms dampens aggregate investment volatility (a 0.03% decline compared to 0.45% in a representative firm benchmark) but amplifies fluctuations in consumption and output. This amplification arises because muted investment cuts reduce the pool of domestic resources available for consumption smoothing, leading to sharper swings in aggregate consumption and output. These dynamics are driven by a novel composition mechanism: constrained firms increase investment when collateral values rebound, offsetting the adjustments of unconstrained firms and reshaping the aggregate response.

*We thank David Domeij, Per Krusell, Lars Ljungqvist and Kurt Mitman for their excellent supervision and generous support.

[†]Stockholm School of Economics. yijia.chen@phdstudent.hhs.se

[‡]IIES, Stockholm University. idakristine.haavi@iies.su.se

1 Introduction

Global financial integration has accelerated over the past few decades. Many emerging and advanced economies have deregulated capital controls and shifted from fixed to floating exchange rate regimes. As a result, foreign capital inflows have surged, and private external borrowing has risen sharply, fueling increases in investment and consumption. However, the Global Financial Crisis of 2008 and the subsequent eurozone crisis exposed the vulnerabilities of this arrangement: even modest reversals in external funding can trigger credit tightening, sharp contractions in investment, and, in extreme cases, currency and sovereign debt crises. These episodes have sparked renewed interest in understanding how reversals of foreign capital flows propagate through a small open economy and their implications for welfare.

This paper contributes to this understanding by analyzing how a sudden reversal in foreign capital flows affects firm decisions and, in turn, aggregate consumption, output, and investment in a small open economy. We study a setting with heterogeneous firms subject to financial frictions, in which a representative household can borrow and save in both domestic and foreign currencies. To this end, we embed firm-level financial frictions into a small open economy New Keynesian model with heterogeneous firms. Each firm faces two constraints: (i) a nonnegativity constraint on dividends and (ii) a collateral constraint on borrowing. Markets clear under a flexible exchange rate, and agents have perfect foresight regarding the path of capital inflows. We calibrate the model to match key moments of the Hungarian economy just prior to the Global Financial Crisis and then introduce a hypothetical foreign-borrowing shock: an unanticipated 1% drop in external lending (equivalent to about 0.1% of steady-state GDP), followed by a gradual mean-reverting recovery.

By examining this illustrative foreign-borrowing shock, we shed light on the interaction between firm-level borrowing constraints and capital flow dynamics. Our analysis offers three key contributions. First, we extend the Sudden Stop literature by integrating collateral constraints into a small open economy Heterogeneous Agent New Keynesian (HANK) framework, showing how even a modest hypothetical shock can generate rich aggregate dynamics.

Second, we demonstrate that financial frictions and firm heterogeneity amplify fluctuations in aggregate consumption, not by deepening the investment contraction, but precisely by moderating it. This moderation reduces the pool of domestic resources that can shift toward consumption. Third, we provide a quantitative assessment of how the proportion of financially constrained firms shapes the magnitudes of fluctuations in investment, consumption, and output, offering policy-relevant insights for countries when designing macroprudential policies and exchange rate regimes.

Our main finding is that the distribution of firms with respect to their financial constraints is crucial to understanding the economy's response to reversals in foreign capital flows. Specifically, we find that aggregate investment volatility is dampened in the heterogeneous firm model compared to a representative firm model. In the representative firm case, investment falls by 0.45% in the first period, while in the heterogeneous firm model, where 59.25% of firms are collateral-constrained, it declines by only 0.03%. Interestingly, this muted investment response amplifies the drop in consumption and the increase in output. In the representative firm scenario, a large reduction in investment frees up domestic resources, labor, and goods, thereby dampening the decrease in consumption (which falls by only 0.04%) and limiting the rise in output (to 0.01%). By contrast, when investment barely adjusts, fewer resources are reallocated to consumption, causing a sharper drop in consumption (0.08%) and a more pronounced rise in output (0.05%).

1.1 Related literature

This paper contributes to three strands of literature. First, it builds on work examining how aggregate shocks affect firm investment, particularly in the presence of financial frictions. Our model draws heavily on Khan and Thomas (2013), Ottonello and Winberry (2020), and Khan et al. (2020), all of which study how unanticipated shocks impact firm behavior under different forms of credit constraints. Ottonello and Winberry develop a heterogeneous firm New Keynesian model in which firms face idiosyncratic productivity shocks and make investment and financing decisions by issuing bonds. Crucially, firms may default on their debt, which leads to endogenous discounting based on default risk. They show that monetary policy shocks disproportionately affect low-risk (i.e., less financially constrained) firms, highlighting the importance of the firm distribution for the transmission of policy shocks. We extend this framework to an open economy setting in which firms face collateral constraints rather than default risk. Instead of analyzing domestic monetary policy, we study a negative shock to foreign capital inflows. Similarly to Ottonello and Winberry, we find that the cross-sectional distribution of financial constraints is central to understanding the aggregate effects of the shock. We also use a collateral constraint in a heterogeneous firm setting, rather than debt default. Ottonello and Winberry (2024) use this to study how this impacts firms' choice between investing in capital and innovation. Jo and Senga (2019) look at how subsidies targeted at more constrained firms increases aggregate productivity by reducing misallocation. Instead, we study the presence of collateral constraints changes the response to a shock to foreign capital inflow.

Second, the Sudden Stop literature has long emphasized the role of collateral constraints in amplifying external-funding reversals. Kehoe and McGrattan (2005) show that sudden stops and financial frictions cannot generate observed output drops by themselves, but that these, in combination with policy distortions inducing productivity declines, are necessary. Arellano (2008) shows how endogenous government debt default leads to sudden stops and generates high volatility in consumption due to limited insurance in these economies. Mendoza (2010) shows that endogenous borrowing limits generate debt-deflation dynamics that exacerbate output contractions when foreign capital inflows abruptly reverse.Fornaro (2015) has a similar model but highlights that financial frictions create a pecuniary externality where there is a choice between price stability and supporting asset prices. During sudden stop episodes, a flexible exchange rate helps to increase collateral values and thereby dampening the negative effects on consumption and investment volatility. Villalvazo (2024) shows in a similar setting with debt-deflation dynamics how heterogeneity among households matters for aggregate effects as more leveraged households substantially reduce consumption, showing that higher inequality leads to more severe crises with stronger and more persistent current account reversals. Schmitt-Grohé and Uribe (2021) demonstrate that collateral value feedback can even produce self-fulfilling Sudden Stop equilibria. Building on these insights, this paper embeds potentially binding collateral constraints directly on firms' borrowing and quantifies how the cross-sectional distribution of these constraints governs the propagation of foreign shocks. Unlike these papers, we find that the presence of constrained firms mutes the investment response to even a small, 1 percent drop in external lending.

Finally, in parallel, a growing body of work has extended HANK models to open economies, highlighting distributional propagation channels. Sunel (2018) documents how gradual disinflation in an small open economy with incomplete household markets yields regressive welfare effects. Zhou (2022) finds that the covariance between households' marginal propensities to consume and their dollar-debt exposures drives aggregate consumption volatility under exchange-rate shocks. Verner and Gyöngyösi (2020) show that when households hold debt denominated in foreign currency, following a local currency depreciation, reduce consumption one-to-one with their debt revaluation. De Ferra et al. (2020) further integrate incomplete markets and price rigidity to quantify how household balance-sheet exposures amplify Sudden Stop dynamics. Auclert et al. (2021) show that heterogeneity amplifies the real-income channel of exchange-rate movements, potentially making depreciations contractionary. Iyer (2015) shows that in a setting with both wealthy households participating in the asset market, and hand-to-mouth households, the optimality of monetary policy under an exchange rate peg is increasing in the share of hand-to-mouth households. This is because real exchange rate fluctuations gives rise to a high consumption volatility for these households. Ottonello (2021) show that in a setting with both collateral constraints on foreign currency borrowing, and nominal wage rigidities, policy-makers face a tradeoff between reducing unemployment and upholding collateral values following external shocks. Oskolkov (2023) show that floating exchange rate regimes poorer households through dampening interest hikes. Guo et al. (2023) show that household integration in international trade and financial markets is crucial to understanding aggregate effects of external shocks. Cugat (2019) shows that household heterogeneity in uninsurable income risk, and in wealth, amplifies consumption and output drops following external shocks. Drenik et al. (2018) show that wealthier households are more likely to hold assets denominated in foreign currency, leading to heterogeneous exposure to exchange rate movements. A home currency depreciation then increases wealth inequality, showing policy should account for the distributional effects. This paper instead focuses on firm balance sheets, embedding both dividend and collateral constraints into a small open economy HANK setup to trace how firm heterogeneity and financial frictions reshape the dynamics of investment, consumption, and output following reversals in foreign borrowing. There are also papers that study settings where firms borrow directly in foreign currency. Aghion et al. (2004) show that firms optimally borrow in foreign currency, and that with credit constraints, credit easing can help dampen firms' credit crunches in a credit crisis. Salomao and Varela (2022) show that when more productive and high propensity to invest type firms issue foreign currency debt, aggregate investment increases, and default rates decreases. We instead focus on a setting where foreign borrowing effects firms indirectly through terms of trade conditions and demand.

By uniting the Sudden Stop and small open economy strands of the HANK literature, this framework uncovers a novel composition mechanism. When foreign borrowing contracts, the majority of collateral-constrained firms, once collateral values rebound, expand investment, muting the aggregate investment drop. At the same time, muted investment cuts leave fewer domestic resources (labor and goods) for consumption, thereby amplifying the consumption decrease and output surge relative to a representative firm benchmark. Embedding firm-level frictions into a HANK setting thus reveals how even modest external shocks can generate highly nonlinear macroeconomic fluctuations.

The paper is organized as follows. Section 2 describes the model. Section 3 describes our calibration and computational methods. In section 4 we move on to a discussion of our results, while section 5 concludes.

2 Model

In this section, we describe the model in detail. The model consists of three blocks: an investment block populated by a distribution of heterogeneous firms facing financial frictions, a New Keynesian block featuring price rigidities, and a representative household who saves and borrows foreign capital, and consumes a mix of home and foreign final goods.[1] The model is built on the one developed by Ottonello and Winberry (2020). We extend their framework to an open economy setting with free flows of goods and capital between home and foreign.

2.1 Preliminaries

The model features two countries, Home and Foreign, where Home is a small open economy taking world market prices and foreign capital inflows as given. We assume that purchasing power parity holds between the two countries:

$$P_{H,t} = e_t P^{\star}_{H,t}, \quad P_{F,t} = e_t P^{\star}_{F,t} \tag{1}$$

where e_t is the exchange rate between home currency and foreign currency, defined as the price of one unit of foreign currency in terms of home currency in period t. Therefore, an increase in e_t indicates the depreciation of the domestic currency against the foreign currency. For simplicity, we normalize $P^{\star}_{F,t} = 1$, such that $P_{F,t} = e_t$.

[1] The international market's decisions are all taken as exogenous in our model, and only enters the model through the household's decisions.

2.2 Investment block

There is a $[0, 1]$ continuum of production firms that are incumbents or entrants. Firms produce undifferentiated intermediate goods with a decreasing returns-to-scale production function: $y = zk^{\alpha}l^{\nu}$, with $\alpha + \nu < 1$. Capital k and labor l are the only two inputs, and z is the level of stochastic idiosyncratic productivity of the firm. They invest in capital subject to financial frictions. They have two available instruments to finance investments. First, firms can use their cash on hand, subject to a nonnegative dividend constraint.[2] Second, firms can issue bonds subject to a collateral constraint. Production firms differ in three dimensions: idiosyncratic productivity z, capital stock k, and real debt position b.[3] All production firms are owned by the representative household, and there is no aggregate uncertainty.

Production Firms. At the beginning of each period, each firm is hit by an idiosyncratic productivity shock, which follows a log-AR(1) process denoted by $H(z'|z)$:

$$\log z' = \rho_z \log z + \epsilon', \quad \epsilon \sim \mathrm{N}(0, \sigma_z^2)$$

After firms observe their productivity level, they make their labor decisions to maximize profits. A firm's profit ι_t and cash on hand n_t are defined as follow:

$$\iota_t(z, k, b) = \max_{l} \; p_t y - w_t l$$

$$n_t(z, k, b) = \iota_t(z, k, b) + q_t(1 - \delta)k - \frac{b}{\Pi_{H,t}}$$

where p_t is the real price of undifferentiated intermediate goods relative to the final home goods, w_t is real wages, δ is the capital depreciation rate, q_t is the real price of capital, and $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$ is the inflation of home final goods. Note that the firm's optimal labor decision

[2]We assume that firms cannot issue equity to finance investments, first, because the literature has documented significant costs to equity issuance, and relatedly, equity issuance is very infrequent.

[3]The debt position of a firm is positive when the firm is a net debtor, and is negative when the firm is a net saver.

and maximized profits are given by:

$$\begin{aligned} l^\star &= \left(\frac{p_t \nu z k^\alpha}{w_t}\right)^{\frac{1}{1-\nu}} \\ \hat{\iota}_t(z,k,b) &= (p_t z)^{\frac{1}{1-\nu}} \left(\frac{\nu}{w_t}\right)^{\frac{\nu}{1-\nu}} k^{\frac{\alpha}{1-\nu}} \end{aligned} \tag{2}$$

Once a firm completes operations and repays all its debt, they receive an exogenous exit shock with probability π_d, preventing the firm from continuing into the next period. Conditional on receiving an exit shock, the firm rebates all profits to the representative household as dividends and leaves the economy. Firms that are not hit by the exit shock, then make investment and borrowing decisions prior to continuing to the next period, by their flow profit function:

$$F_t(z,k,b) = n_t(z,k,b) - q_t k' + Q_t b', \tag{3}$$

where q_t is the price of capital, k' is the firm's next period capital stock, Q_t the real debt discount rate, and b' the firm's next period's debt.

The value function that incumbent firms solve is then:

$$V_t(z,k,b) = \pi_d n_t(z,k,b) + (1-\pi_d)\tilde{V}_t(z,k,b), \tag{4}$$

$$\tilde{V}_t(z,k,b) = \max_{k',b'} F_t(z,k,b) + \mathbb{E}_t\left[\left(\frac{x_{t+1}}{x_t}\right)^{1-\chi} \Lambda_{t+1} V_{t+1}(z',k',b') \Big| z\right] \tag{5}$$

$$\text{s.t.} \quad F_t(z,k,b) \geq 0, \qquad \frac{b'}{\Pi_{H,t+1}} \leq \lambda q_{t+1} k' \tag{6}$$

where $x_t = \frac{P_{H,t}}{e_t}$ is the terms of trade, and $\Lambda_{t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)}$ the real stochastic discount factor of the representative household.[4]

In equation (5), we see that firms choose next period capital k' and debt b', to maximize their continuation value $\tilde{V}_t(z,k,b)$, subject to the two constraints in (6). The first constraint is a nonnegativity constraint on dividends, meaning that firms cannot finance investments

[4]Firms have the same stochastic discount factor as the representative household because the representative household owns all firms.

through equity issuance.[5] Whenever flow profits are positive, this will be rebated to the representative household at the end of the period. The second constraint is a collateral constraint, where the real value of a firm's debt should not exceed a certain $\lambda \in (0,1)$ fraction of the firm's capital value. The collateral constraint mimics the financial friction that limits small- and medium-sized firms' borrowing capacity.[6]

Importantly, note that a firm's value is affected by a change in the terms of trade $x_t = \frac{P_{H,t}}{e_t}$ of the home economy, even though the terms of trade does not enter the firm's profit function $\iota_t(z,k,b)$ directly. This is related to two features of our model. First, because final goods are traded on the international market, any change in the terms of trade shifts relative prices. In turn, this changes the relative demand of final goods from home and foreign, by both home and the foreign consumers. This indirectly changes the demand for intermediate goods used to produce the home final good, and thereby the firm's value function. Second, changes in the terms of trade affects the household's aggregate consumption, such that her stochastic discount factor changes, changing the value for all firms. This explains why the variation in terms of trade $\frac{x_{t+1}}{x_t}$ enters the firm's continuation value in Equation (5).

Entry and exit of firms. Due to the exit shock, a measure π_d of incumbent firms exogenously exits the economy after production in each period. At the beginning of each period, we assume that there is an equal mass of firms entering production such that the total mass of firms remains time-invariant. Entering firms differ in their initial productivity z_0, which they draw from a time-invariant distribution $G(z)$: $z_0 \sim G(z)$, $G(z) = \log \mathrm{N}(0, \frac{\sigma_z^2}{1-\rho_z})$. They receive initial capital $k_0 > 0$ from the representative household, and their initial debt holdings are zero: $b_0 = 0$. Entering firms proceed to produce, and make decisions in the same way as incumbent firms.

We can now define the state space of firms, denoted by $(Z \times K \times B) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$ denote the state space of firms. Define a measure $\mu_t(z,k,b) : (Z \times K \times B) \to [0,1]$. The

[5]We see this as a reasonable constraint, as equity issuance is expensive for firms, hence they do this very infrequently. This increases the computational efficiency of the model in general equilibrium.

[6]This mimics the case where firms lack commitment, banks get a fraction of firms' collateral when they default, or banks offer only risk-free loan contracts.

transition of beginning-of-period firm distribution is characterized by the following equation:

$$\begin{aligned} \mu_t(Z \times K \times B) &= \mu_{t-1}^{stay}(Z \times K \times B) + \mu_t^{ent}(Z \times K \times B) \\ \mu_{t-1}^{stay}(Z \times K \times B) &= \int_k \int_b \int_z \mu_{t-1}^{stay}(z \times k' \times b') \; dH(z'|z) \; db \; dk \\ \mu_t^{ent}(Z \times K \times B) &= \int_k \int_b \pi_d \int_z \mu_t^{ent}(z, k_0, b_0) \; dG(z) \; db \; dk \end{aligned} \tag{7}$$

where $\mu_t^{stay}(Z \times K \times B)$ denotes the measure of firms who are not affected by the exogenous exit shock in period t, and $\mu_t^{ent}(Z \times K \times B)$ denotes the measure of entering firms in period t. The measure of all firms $\mu_t(Z \times K \times B)$ equals one, as every period a π_d measure of firms leaves the economy due to the exit shock, while another π_d measure of firms enters. In steady state, while firms' productivity level changes over time due to idiosyncratic productivity shocks and firms change their investment and debt positions accordingly, the cross section of firms remains time-invariant.

2.3 New Keynesian Block

Retailers and Final Good Producer. We have a unit measure of retailers. They purchase undifferentiated intermediate goods from production firms and use them as input to produce differentiated goods. Retailers produce with a linear one-to-one production technology: $\tilde{y}_{it} = y_{it}$, where $\tilde{y}_{it}$ is the differentiated good produced by retailer i and y_{it} is the undifferentiated intermediate good demanded by retailer i. The market for differentiated goods is under monopolistic competition, and the elasticity of substitution between retailers' differentiated goods is ϵ. Each retailer sets period t nominal price $\tilde{p}_{it}$ to maximize its present discounted value of profits, and they are subject to a fixed real adjustment cost $\frac{\varphi}{2}(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1)^2 Y_t$:

$$\max_{\{\tilde{p}_{it+s}\}_{s=0}^{\infty}} \; \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+1} \frac{u'(C_{t+s})}{u'(C_t)} \frac{P_t}{P_{t+s}} \frac{P_{H,t+s}}{P_{H,t}} \left[(\frac{\tilde{p}_{it}}{P_{H,t}} - p_t)(\frac{\tilde{p}_{it}}{P_{H,t}})^{-\epsilon} Y_t - \frac{\varphi}{2}(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1)^2 Y_t \right]$$

From the retailer's problem we can derive the linearized New Keynesian Phillips Curve (NKPC)[7]:

$$\log \Pi_{H,t} = \frac{\epsilon - 1}{\varphi} \log(\frac{p_t}{p^{ss}}) + \beta \mathbb{E}_t \log \Pi_{H,t+1} \tag{8}$$

where p^{ss} is the steady state value of p_t. Y_t is the final good, produced by a representative final good producer with a CES production function, aggregating differentiated goods from the retailers, $Y_t = (\int_0^1 \tilde{y}_{it}^{\frac{\epsilon-1}{\epsilon}} \, di)^{\frac{\epsilon}{\epsilon-1}}$. Hence, the final good producer solves the following problem:

$$\max_{\tilde{y}_{it}} \quad P_{H,t} \left(\int_0^1 \tilde{y}_{it}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 \tilde{p}_{it} \tilde{y}_{it} \, di$$

Home final goods can be consumed by the representative household, exported to foreign consumers through international trade, or used to produce capital goods.

Capital Good Producers. There is a unit measure of competitive capital good producers who purchase final goods from the final good producer, produce capital goods, and sell them to production firms at price q_t. Their production technology is one-for-one, and is subject to a convex adjustment cost $\Phi(\frac{I_t}{K_t})K_t = \frac{\phi}{2}(\frac{I_t}{K_t} - \tilde{\delta})^2 K_t$. $\tilde{\delta}$ is the steady state investment rate. Each period, new investment equals the sum of capital demanded by entering firms and operating incumbent firms, less the aggregate undepreciated capital stock:

$$I_t = \int [k' - (1-\delta)k] \, d\mu_t^{stay} + \int k_0 \, d\mu_t^{ent} - (1-\delta) \int k d\mu_t^{exit} \tag{9}$$

where μ_t^{exit} denotes the mass of firms that leave the economy in period t.

A capital good producer then solves the following problem:

$$\max_{I_t} \quad q_t I_t - [I_t + \Phi(\frac{I_t}{K_t})K_t]$$

[7]For derivation of the linearized NKPC see Appendix A.

From its optimality condition, we can derive the pricing equation for capital:

$$q_t = 1 + \phi(\frac{I_t}{K_t} - \tilde{\delta}) \tag{10}$$

Monetary authority. The central bank sets the interest rate to achieve the flexible price allocation in the Home economy, allowing the exchange rate to fluctuate. This means that the nominal price of the home final good $P_{H,t}$ is constant in equilibrium, such that nominal price rigidities have no impact on the real economy:

$$P_{H,t} = P_H, \quad \forall t \tag{11}$$

2.4 Representative Household and the International Market

Representative Household. There is a representative household in the home economy. It derives utility by consuming both home final goods and imported foreign goods, supplies labor to production firms, and makes consumption and saving decisions:

$$\max_{C_t, L_t} \; E_0 \sum_{t=0}^{\infty} \beta^t \Big(\log C_t - \Psi L_t \Big) \tag{12}$$

$$\text{s.t.} \quad \frac{P_t C_t}{P_{H,t}} + \tilde{B}^{HH}_{t+1} + q_t \int k_0 \; d\mu_t^{\text{entrant}} = (1 + r_t)\tilde{B}^{HH}_t + w_t L_t$$
$$+ \int_{d=0} F_t(z, k, b) \; d\mu_t + \pi_t^{\text{Retailer}} + \pi_t^{\text{Capital}} + q_t(1 - \delta) \int k \; d\mu_t^{\text{exit}}$$

C_t is the household's aggregate consumption, $\tilde{B}^{HH}_t$ the household's real asset, denominated in the home good, L_t the household's labor supply, and Ψ is a parameter governing labor disutility. In period t, the representative household exchanges $\tilde{B}^{HH}_{t+1}$ units of real bonds with a mass of financial intermediaries, and each real bond pays $1 + r_{t+1}$ units of home final goods in the next period. The representative household also invests initial capital k_0 in each new entering firm in exchange for its ownership and claim to dividends. We assume that the representative household owns all production firms and all retailers, which is why firms and households share the same stochastic discount factor. Production firms' flow profits, F_t, are rebated to the household as dividends. Similarly, dividends from the retailers are denoted by π_t^{Ret}.

With trade in goods, consumption C is made up of two components: final home goods C_H and final foreign goods C_F. The representative household optimally chooses C_H and C_F by solving the following problem:

$$\max_{C_H, C_F} \; C = \left[(\chi)^{\frac{1}{\theta}} (C_H)^{\frac{\theta-1}{\theta}} + (1-\chi)^{\frac{1}{\theta}} (C_F)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

$$\text{s.t.} \quad P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$$

where θ is the elasticity of substitution between home final good and foreign final good. From the household's problem we can solve for the aggregate price level P_t:

$$P_t = \left[\chi P_{H,t}^{1-\theta} + (1-\chi) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{13}$$

Since we will calibrate $\theta = 1$, we can derive the following:

$$C = \frac{C_H^{\chi} C_F^{1-\chi}}{\chi^{\chi} (1-\chi)^{1-\chi}} \tag{14}$$

$$P = P_H^{\chi} P_F^{1-\chi} = P_H^{\chi} e^{1-\chi} \tag{15}$$

This implies that, in equilibrium, the household's real consumption expenditures can be decomposed as:

$$\frac{PC}{P_H} = (\frac{1}{x})^{1-\chi} C = C_H + \frac{1}{x} C_F, \tag{16}$$

where $x_t = \frac{P_{H,t}}{e_t}$ is the terms of trade. Then, given $(\frac{1}{x_t})^{1-\chi} C_t$, the household's expenditures on home goods and foreign goods are constant:

$$C_{H,t} = \chi\Big[(\frac{1}{x_t})^{1-\chi}C_t\Big] \tag{17}$$

$$\frac{1}{x_t}C_{F,t} = (1-\chi)\Big[(\frac{1}{x_t})^{1-\chi}C_t\Big] \tag{18}$$

Finally, the household's intratemporal decision is given by:

$$w_t = (\frac{1}{x_t})^{1-\chi}\frac{\Psi}{u'(C_t)} \tag{19}$$

Financial intermediaries. There is a continuum of competitive financial intermediaries in the economy. They receive foreign investment bonds, $\tilde{B}^{\star}$, and purchase/sell bonds to/from all production firms $j \in J$, b_j, and the household, $\tilde{B}^{HH}$. All bonds are real and denominated in the home good. The interest paid on the foreign bond is $r^{\star}$, the interest received on the household bond is r, and financial intermediaries discount bonds issued by firms by Q.

The financial intermediaries per-period real budget constraint must be balanced:

$$\begin{aligned} \int \frac{b_{j,t}}{\Pi_{H,t}}dj &= (1+r_t^{\star})\tilde{B}_t^{\star} + (1+r_t)\tilde{B}_t^{HH} \\ Q_{t+1}\int b_{j,t+1}\ dj &= \tilde{B}_{t+1}^{\star} + \tilde{B}_{t+1}^{HH} \end{aligned} \tag{20}$$

We can obtain the pricing equation for r from the household's problem:

$$\frac{1}{1+r_{t+1}} = \mathbb{E}_t\Big[\Big(\frac{x_{t+1}}{x_t}\Big)^{1-\chi}\Lambda_{t+1}\Big] \tag{21}$$

where $\Lambda_{t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)}$ is the household's real SDF.

To derive the discount factor on bonds issued by firms, let $B_{j,t+1}$ be the nominal face value of an arbitrary corporate bond in the home currency. We assume that these bonds are not inflation-indexed. The real face value of the bond $b_{j,t+1}$ when issued in period t is defined as $b_{j,t+1} = \frac{B_{j,t+1}}{P_{H,t}}$. In the next period, the firm still pays $B_{j,t+1}$, but the real face value will be affected by home good inflation: $\frac{B_{j,t+1}}{P_{H,t+1}} = \frac{P_{H,t}b_{j,t+1}}{P_{H,t+1}} = \frac{b_{j,t+1}}{\Pi_{H,t+1}}$. The financial intermediary lends out $Q_t\tilde{b}_{j,t+1}$ units of home goods and receives $\frac{b_{j,t+1}}{\Pi_{H,t+1}}$. Therefore, it must be that:

$$Q_t = \mathbb{E}_t\left[\frac{1}{\Pi_{H,t+1}}\left(\frac{x_{t+1}}{x_t}\right)^{1-\chi}\Lambda_{t+1}\right]$$

Note that this is the pricing equation for nominal (non-inflation-indexed) bonds, as in Ottonello and Winberry (2020).

The International Market. The international market consists of a goods trade market and a financial market. Let $C^\star_{H,t}$ denote Home's export to Foreign in period t. We assume that Foreign's demand for the Home final good is a decreasing function of Home's terms of trade *vis-à-vis* Foreign:

$$C^\star_{H,t} = x_t^{-\theta^\star} D^\star \tag{22}$$

where $\theta^\star$ is the elasticity of demand from the Foreign household, $D^\star$ is a demand shifter that captures the magnitude of the foreign market, and x_t is the terms of trade as described above.

In addition, in period t, foreign investors inelastically deposit foreign capital $\tilde{B}^\star_{t+1}$ in the Home economy. We assume that this foreign capital inflow is denominated in terms of real domestic good and is fixed in steady state, denoted by $\bar{\tilde{B}}^\star$. In case of an unanticipated shock, the transition of $\tilde{B}^\star_{t+1}$ is governed by the following mean-reverting process:

$$\tilde{B}^\star_{t+1} = (1-\rho_B)\bar{\tilde{B}}^\star + \rho_B\tilde{B}^\star_t + \epsilon^\star_{t+1} \tag{23}$$

We can now combine the markets for goods and capital, to obtain the world market clearing condition:

$$C^\star_{H,t} = \frac{1}{x_t}C_{F,t} + (1+r_t)\tilde{B}^\star_t - \tilde{B}^\star_{t+1} \tag{24}$$

This states that, from the Home economy's perspective, its exports must cover its imports and the net interest payment on foreign capital inflow. This will be important in understanding the transmission of foreign shocks that we analyze in our policy experiment.

2.5 Aggregation and General Equilibrium

We can define aggregate variables as:

$$
\begin{aligned}
Y_t &= \int_{d=0} y_t \; d\mu_t \\
K_t &= \int k \; d\mu_t \\
I_t &= \int [k' - (1-\delta)k] \; d\mu_t^{stay} + \int k_0 \; d\mu_t^{ent} - (1-\delta) \int k d\mu_t^{exit} \\
\Phi_t &= \frac{\phi}{2}(\frac{I_t}{K_t} - \tilde{\delta})^2 K_t \\
\Xi_t &= \frac{\varphi}{2}(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1)^2 Y_t
\end{aligned}
$$

Here, Y_t is aggregate output, K_t is the aggregate capital stock, I_t is aggregate investment, which is equal to incumbents' investment, entrants' initial capital stock, less the capital stock of exiting firms, which is lump-sum rebated to the representative household. Φ_t is the adjustment cost of capital, and Ξ_t is the price adjustment cost.

We define the general equilibrium in this economy as follows: the policy functions $\{k_t'(z,k,b)\}_{t=0}^{\infty}$, $\{b_t'(z,k,b)\}_{t=0}^{\infty}$ and the value functions $\{V_t(z,k,b)\}_{t=0}^{\infty}$, $\{\tilde{V}(z,k,b)\}_{t=0}^{\infty}$ maximize the market value of each production firm; the consumption portfolios $\{C_{H,t}, C_{F,t}\}_{t=0}^{\infty}$ and the aggregate labor supply $\{L_t\}_{t=0}^{\infty}$ maximize the utility of the representative household, given the equilibrium price sequences $\{p_t\}_{t=0}^{\infty}$, $\{\tilde{p}_t\}_{t=0}^{\infty}$, $\{q_t\}_{t=0}^{\infty}$, $\{w_t\}_{t=0}^{\infty}$, $\{r_t\}_{t=0}^{\infty}$, $\{x_t\}_{t=0}^{\infty}$. Prices clear the goods market, the capital market, and the labor market in all periods.

2.6 Classification of firms

In this section, we classified production firms into three groups: Type-1 constrained firms, Type-2 constrained firms, and unconstrained firms. We then discuss their investment and borrowing decisions, and present their distribution in the steady state.

Following Khan and Thomas (2013) and Jo and Senga (2019), we partition the firm distribution into three subsets: unconstrained firms, Type-1 constrained firms and Type-2 constrained firms. First, we define *unconstrained firms* as firms whose nonnegative dividend constraint would not be binding in all possible future states. The Lagrange multipliers for an unconstrained firm's collateral constraint and nonnegative dividend constraint are all zero. An unconstrained firms' investment decision $k_t'^{\star}(z)$ is therefore determined solely by the marginal cost and marginal benefit of investing an additional unit of capital:

$$k_t^{\star}(z) = \arg\max_{k'} \; -q_t k' + \mathbb{E}_t\Big[\Lambda_{t+1}\Big(\text{profit}_{t+1}(z',k') + q_{t+1}(1-\delta)k'\Big)\Big|z\Big] \tag{25}$$

Unconstrained firms are indifferent between any level of b' that leaves them unconstrained. To see this, note that the marginal cost of saving is Q_t in the current period, while the discounted marginal return of saving in the next period is $\frac{1}{\Lambda_{t+1}}$. In equilibrium $Q_t = \frac{1}{\Lambda_{t+1}}$. As long as the firm remains unconstrained, the marginal cost equals the discounted marginal return for any level of b'. To solve this indeterminacy issue, we define the *minimum saving policy* $b^{\star}(z)$:

$$b_t^{\star}(z) = \min_{z'}\Bigg\{\text{profit}_{t+1}(z',k_t^{\star}(z)) + (1-\delta)k_t^{\star}(z) - k_{t+1}^{\star}(z') + Q_{t+1}\min\Big\{b_{t+1}^{\star}(z'),\; \lambda k_{t+1}^{\star}(z')\Big\}\Bigg\} \tag{26}$$

Equation (26) defines a minimum level of firm saving $b_t^{\star}(z)$ that an unconstrained firm could have at the end of period t. By definition, $b_t^{\star}(z)$ ensures that the firm's collateral constraint and nonnegative dividend constraint would not be binding in the next period, given all possible realizations of z'.

In practice, unconstrained firms are characterized by a cash on hand threshold $\bar{n}_t(z)$, all firms with idiosyncratic productivity z and cash on hand $n \geq \bar{n}_t(z)$ are identified as unconstrained firms. To see this, note that when firms make investment and borrowing decisions (k',b'), they only consider two state variables, their idiosyncratic productivity z and their cash on hand n, since there is no real adjustment cost conditional on capital stock

k and debt holding b. Then, the cash on hand n_t of an unconstrained firm investing $k_t^\star(z)$ and saving $b_t^\star(z)$ with nonnegative dividend must satisfy $n - q_t k_t^\star(z) + Q_t b_t^\star(z) \geq 0$, based on which we can define the cash on hand threshold $\bar{n}_t(z)$

$$\bar{n}_t(z) \equiv q_t k_t^\star(z) - Q_t b_t^\star(z) \tag{27}$$

By definition, all firms with $n \geq \bar{n}_t(z)$ will choose $k' = k_t^\star(z)$ and $b' = b_t^\star(z)$, and paying nonnegative dividend $d \geq 0$.

The remaining firms with $n < \bar{n}_t(z)$ are defined as *constrained firms*. Constrained firms can be further classified into two groups: *Type-1 constrained firms* and *Type-2 constrained firms*. For both types of constrained firms, the shadow value of retained earnings is greater than that of paying dividends, therefore they all pay zero dividends and their nonnegative dividend constraint is binding. Type-1 constrained firms adopt the optimal capital policy $k_t^\star(z)$, but cannot adopt the minimum saving policy $b_t^\star(z)$. The debt policy of Type-1 constrained firms can be derived using the nonnegative dividend constrained by substituting $k' = k_t^\star(z)$ and cash on hand n, which is $b' = (q_t k_t^\star(z) - n)/Q_t$. On the other hand, Type-2 constrained firms are facing both a binding collateral constraint and a binding nonnegative dividend constraint, and they can only invest to the extent allowed by the collateral constraint. Their investment and borrowing decisions (k', b') are solutions to the linear system made up by the binding collateral and nonnegative dividend constraints.

The decision rules (k', b') of firms with different states (z, n) are summarized below:

- Firms with $n \geq \bar{n}_t(z)$ are *unconstrained* and adopt $k' = k_t^\star(z)$, $b' = b_t^\star(z)$.
- Firms with $n < \bar{n}_t(z)$ are *constrained*. The upper bound of firm's investment is given by $\bar{k}_t(n) \equiv n/(q_t - \lambda Q_t \Pi_{H,t+1} q_{t+1})$. By comparing $\bar{k}_t(n)$ to $k_t^\star(z)$, we have:
  - Firms with $\bar{k}_t(n) \geq k_t^\star(z)$ are *Type-1 constrained*. They adopt $k' = k_t^\star(z)$, $b' = \frac{q_t k_t^\star(z) - n}{Q_t}$.

– Firms with $\bar{k}_t(n) < k_t^\star(z)$ are *Type-2 constrained.* They adopt $k' = \bar{k}_t(n)$ and $b' = \lambda \Pi_{H,t+1} q_{t+1} \bar{k}_t(n)$.

Figure 1 illustrates the steady-state decision rules for firms with medium levels of idiosyncratic productivity. In the following discussion we remove the time subscript since the firm's problem is time-invariant in steady state. The left panel shows how such firms choose capital, debt, and dividends when they have limited cash on hand. In this case, the firm cannot fully finance its optimal investment level, denoted by $k^\star(z)$, using internal funds alone and must rely on external borrowing. However, due to limited commitment, borrowing is constrained by the amount of capital investment, which serves as collateral. Under these conditions, the firm is classified as Type-2 constrained, meaning its investment and borrowing decisions are interdependent. A Type-2 firm invests less than the optimal $k^\star(z)$, using all available cash and the maximum allowable debt to fund investment, and pays no dividends to the representative household.

Assuming the firm's idiosyncratic productivity remains unchanged, its profits are gradually added to its cash holdings. As internal funds grow, the firm increases both investment and borrowing, leading to further cash accumulation in subsequent periods. However, because the production function exhibits decreasing returns to scale, the firm eventually reaches the optimal capital level $k^\star(z)$, where the marginal return on investment equals its cost. At this point, investment stops increasing, and the firm begins deleveraging by reducing its debt. Once it reaches the optimal saving policy threshold $b^\star(z)$, it ceases to accumulate savings and begins to pay out dividends to the representative household.

3 Calibration

We calibrate our model to the Hungarian economy. Since we do not have access to Hungarian firm-level data, we imputed most of the parameter values from the previous literature that studies firm dynamics in Hungary. The remaining parameters are jointly calibrated to match

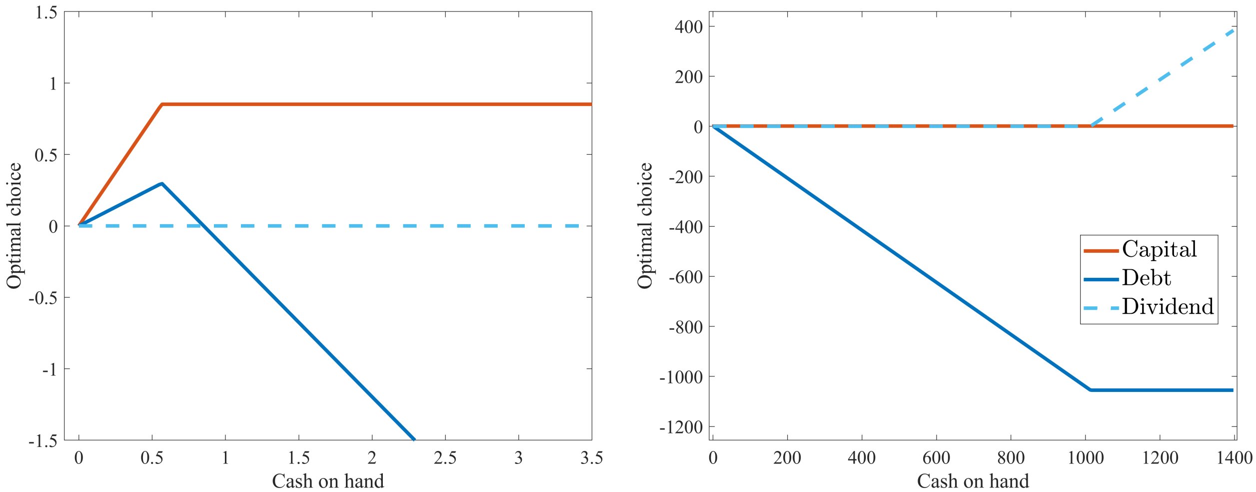

Figure 1: Firm decision rules

selected moments of Hungarian firms. Time is discrete, and we calibrate our model at an annual frequency.

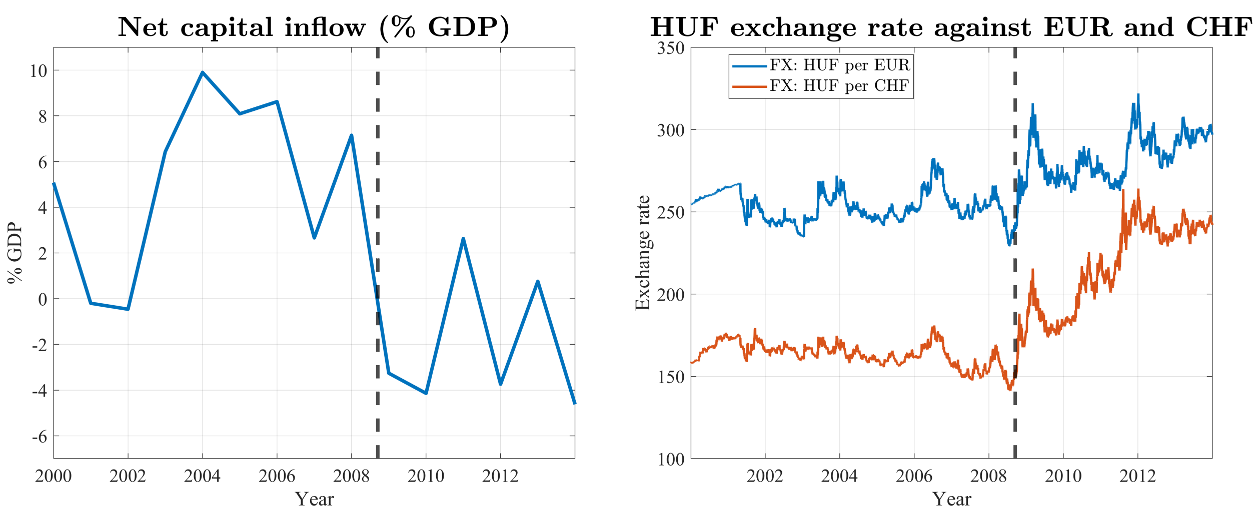

Figure 2: Reverse of foreign capital inflow in Hungary after the 2008 financial crisis. Left panel: annual net foreign capital inflow as % of Hungary GDP. Right panel: the exchange rate of Hungarian forint (HUF) against Euro (EUR) and Swiss Franc (CHF). The black dashed lines correspond to September 15th, 2008, the date when Lehman Brothers collapsed.

Our calibration of the corporate sector relies on Kátay and Wolf (2008) and Salomao and Varela (2022). Kátay and Wolf (2008) estimated a Cobb-Douglas production function using the balance sheet data of Hungarian firms over 1993-2004. Following the estimation strategy suggested by Levinsohn and Petrin (2003), they concluded that in Hungary the

Table 1: Calibration

Parameter	Value	Target/Source
Discount factor	$\beta = 0.96$	Real interest rate, 4.1%
Disutility of labor	$\Psi = 1.06$	Employment share, 50%
Intertemporal EOS of Household	$\sigma = 1$	De Ferra et al. (2020)
Home and foreign good EOS in home	$\theta = 1$	Feenstra et al. (2018)
Home and foreign good EOS in foreign	$\theta^{\star} = 3$	Feenstra et al. (2018)
Share of home goods in consumption	$\chi = 0.6$	De Ferra et al. (2020)
Capital share	$\alpha = 0.29$	Kátay and Wolf (2008)
Labor share	$\nu = 0.50$	
Firms' productivity	$\rho_z = 0.63$	
	$\sigma_z = 0.57$	Salomao and Varela (2022)
Depreciation rate	$\delta = 0.1$	Salomao and Varela (2022)
Entrant's initial capital	$k_0 = 0.003$	Entrant's employment share, 3%
EOS between home varieties	$\epsilon = 10$	De Ferra et al. (2020)
Capital adjustment cost parameter	$\phi = 17$	De Ferra et al. (2020)
Rotemberg adjustment cost parameter	$\varphi = 100$	Kaplan et al. (2018)
Supply of foreign credit	$\bar{B}^{\star} = 0.11$	Private inflow (% GDP), 9.58%
Foreign demand shifter	$D^{\star} = 0.39$	

labor share is around $\nu = 0.5$, and the capital share is around $\alpha = 0.29$. Salomao and Varela (2022) estimated Hungarian firms' productivity process using Hungarian firms' balance sheet data and banks' corporate credit data. They followed Bloom et al. (2018) and Gopinath et al. (2017) and ran regressions. Their estimation result suggested that the persistence of Hungarian firms' productivity is $\rho_z = 0.63$, and the standard deviation of productivity shock is $\sigma_z = 0.57$. Salomao and Varela (2022) also reports a $\delta = 10\%$ capital depreciation rate of the Hungarian economy, which we adopt directly. We also must calibrate the initial capital holding of new entrant firms. Lacking firm-level data to pin down this value directly, we instead target the entrants' share of total employment. Following Ottonello and Winberry (2020), we set the employment share of the entrants to 3% and then choose the initial capital level k_0 so that the model reproduces this moment. Solving for k_0 yields a value of 0.003.

We then calibrate parameters that are relevant to the nominal rigidity. Following De Ferra et al. (2020), the elasticity of substitution between differentiated goods is set to be $\epsilon = 10$, so that the steady state markup is 10% of the output. Also, De Ferra et al. (2020) recommend setting the capital adjustment cost parameter to $\phi = 17$, to match the elasticity of investment to Tobin's Q in Hungary. Following Kaplan et al. (2018), the Rotemberg price adjustment cost is set to $\varphi = 100$.

As for the representative household, we set household's discount factor $\beta = 0.96$, matching an average of 4.1% real interest rate over 2001-2008. We set the elasticity of intertemporal substitution $\sigma = 1$. Labor disutility Ψ is calibrated so that in steady state the representative household's aggregate labor supply is 0.5, matching the average labor participation rate in Hungary from 2001 to 2008. Following De Ferra et al. (2020), the share of home goods in consumption is set to be $\chi = 0.6$, so that the ratio of exported value added to GDP is around 33%. According to Feenstra et al. (2018), the elasticity of home and foreign goods in Home is set to be $\theta = 1$, while the elasticity of home and foreign goods in Foreign is set to be $\theta^{\star} = 3$.

Finally, we assume that, in steady state, Foreign investor's lending to Home is approximately 9.58% of Foreign's output, matching an average of 9.58% of private inflows excluding direct investment (% of GDP) in Hungary, over 2001-2008.

The calibrated parameters are summarized in Table 1.

3.1 Stationary Distribution

The numerical results given our calibration show a stationary distribution in which only a small fraction of firms are financially unconstrained. As shown in Figure 3, the distribution of firms' cash on hand is highly skewed, with a large concentration near zero. This reflects the prevalence of liquidity constraints among firms in the economy. Our numerical results indicate that 59.25% of firms are Type-2 constrained, 40.53% are Type-1 constrained, and only 0.22% operate without financial constraints. The predominance of constrained firms plays a critical role in determining the economy's response to fluctuations in foreign capital

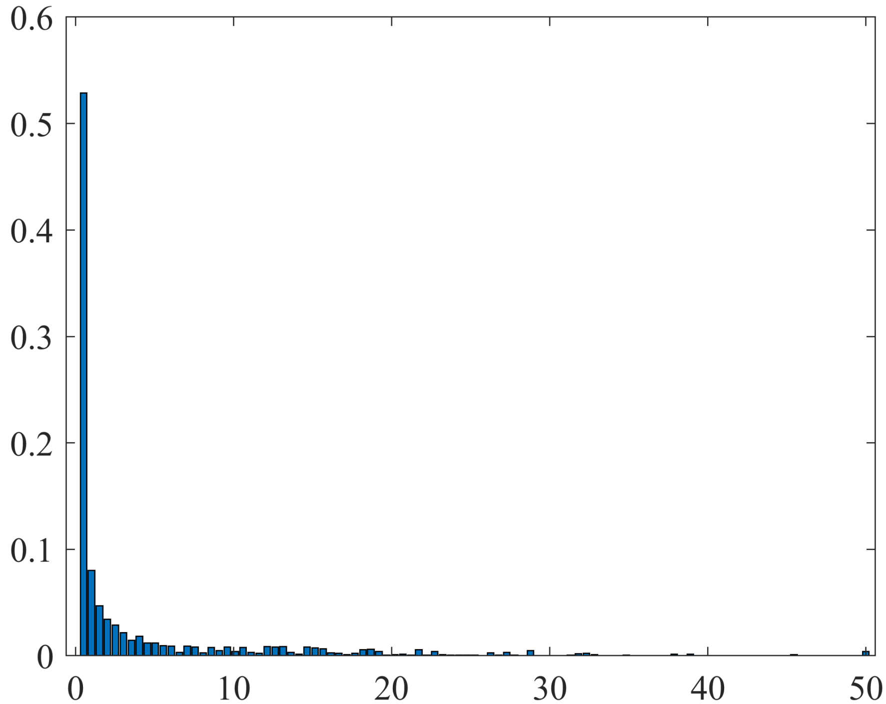

Figure 3: Histogram of firm's cash on hand in the steady state

flows, as we will explore in the next section.

4 Experiment: An Unanticipated Foreign Capital Outflow

In this section, we present a numerical experiment in which the home economy experiences an unanticipated, temporary capital outflow. We begin by plotting the transition dynamics of the benchmark model with heterogeneous firms. Next, we compare these results with those from a frictionless model, which—due to the absence of financial constraints—collapses to a representative firm framework. Finally, we examine the impulse responses of different types of firms in the benchmark model to shed light on the mechanisms driving the divergence between the heterogeneous firm economy and its representative firm counterpart.

4.1 Transitional dynamics

We conduct the following experiment: in period 1, the foreign capital inflow $B^{\star}$ experiences an unanticipated 1% decline, which corresponds to approximately 0.1% of steady-state output. After this initial shock, capital inflows gradually recover with a persistence parameter of $\rho_B = 0.5$. Given this path, we compute the impulse responses of prices and key aggregate variables to trace the economy's dynamic adjustment.

This experiment represents our main scenario: a sudden stop in foreign capital inflows followed by a perfect-foresight transition back to the steady state. It is designed to capture the key features of Hungary's experience during the Global Financial Crisis in 2008, when a sharp contraction in external financing led to a rapid depreciation of the Hungarian forint.

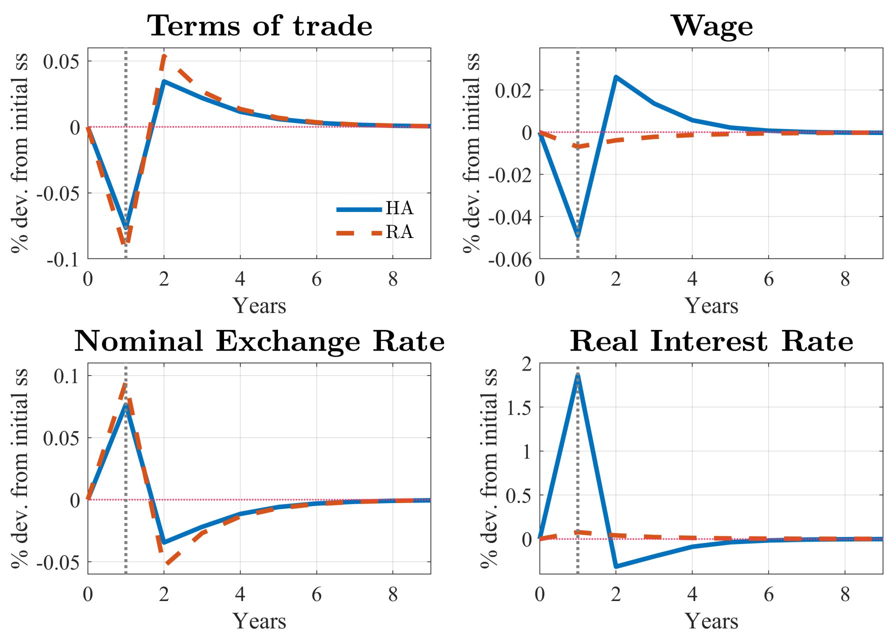

Figure 4: Evolution of prices after an unanticipated shock on foreign capital outflow

The solid blue lines in Figures 4 and 5 depict the dynamic responses of aggregate variables and prices following the capital outflow shock. The initial impact is felt through the balance of payments: with reduced access to foreign capital, the home economy faces a higher net external payment. This necessitates a reallocation of domestic resources to meet external obligations. In response, market-clearing prices—including the real interest

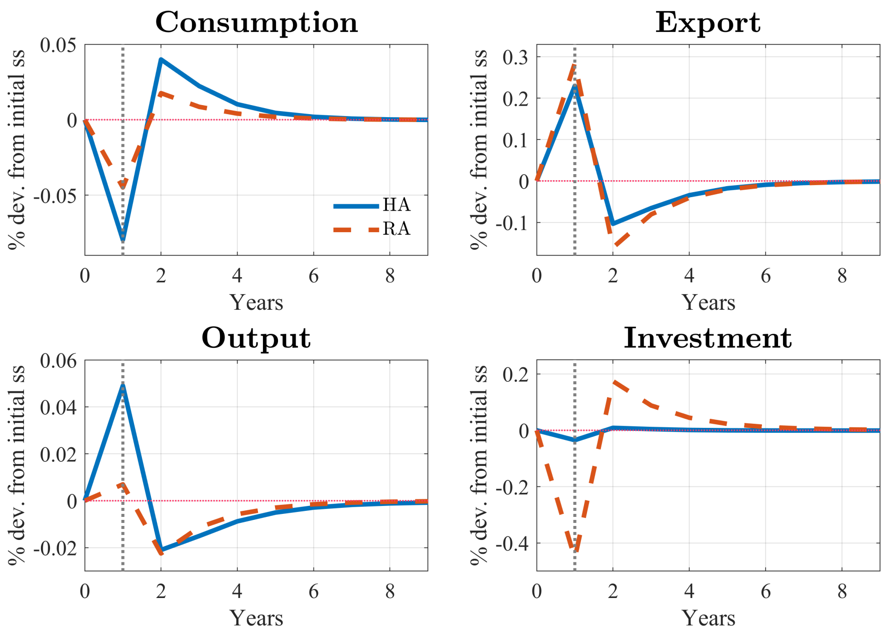

Figure 5: Impulse responses of aggregate variables after an unanticipated shock on foreign capital outflow

rate r_t, wage rate w_t, and terms of trade x_t—adjust accordingly, which in turn alters firms' investment and borrowing decisions.

In the first period, the terms of trade deteriorate, boosting exports and helping to offset the increased net external payment. Under the assumed flexible exchange rate regime, the domestic currency depreciates while the price of home goods remains fixed in domestic terms. The worsening terms of trade also compel households to reduce their imports. Meanwhile, falling wages incentivize firms to expand labor demand. Because capital stocks are predetermined, the increased labor input raises firms' output. However, this output expansion does not lead to a corresponding rise in consumption. Instead, the real interest rate—measured in domestic goods—rises, encouraging households to cut consumption of both domestic and foreign goods. As a result, net exports rise. At the same time, the higher real interest rate discourages firm investment, resulting in a decline in aggregate investment during the first period.

From the second period onward, the direction of foreign capital flows reverses. As capital returns to the home economy, net external payments fall below their steady-state level, implying that the economy no longer needs to export as many domestic goods to finance imports and service external obligations. This reversal is reflected in price movements: the terms of trade, wage rate, and real interest rate all begin to move in the opposite direction. These price adjustments, in turn, induce a reversal in the dynamics of aggregate variables.

In response to the shock to foreign capital inflow and associated price changes, households increase their consumption. At the same time, firms reduce labor demand, produce fewer domestic goods, and raise investment levels. Subsequently, both prices and aggregate quantities gradually converge back to their steady-state values.

4.2 Comparison to the case with no financial frictions

To understand the role of financial frictions, we compare our model to a setting with no financial frictions. We do this by eliminating both financial frictions in our economy, that is, the nonnegativity constraint on dividends and the collateral constraint.[8] With no financial frictions, all firms are free to choose the unconstrained policies $k'^{\star}(z)$. As is shown in *Appendix B* of Khan and Thomas (2008), a heterogeneous firm problem with no financial frictions can be written as a representative firm problem. This is because the aggregate capital choice now in a separable way depends on the firm-specific productivity and the aggregate state, so that the type-specific capital choice is time-invariant. In the representative firm economy, we use the same percentage deviation in foreign capital inflows as in the benchmark heterogeneous firm case, although the absolute values may differ. Based on this shock path, we compute the impulse responses of aggregate variables and prices.

The dashed red lines in Figures 4 and 5 show the results from the representative firm model. A key difference that emerges is the significantly higher volatility of aggregate investment. In the first period, following the capital flow shock, investment in the representative firm economy drops by 0.45%, while consumption falls by only 0.04% and output increases slightly by 0.01%. In contrast, in the benchmark heterogeneous firm economy, investment

[8] With no nonnegativity constraint on dividends, the collateral constraint becomes superfluous, as firms can just issue equity instead of bonds.

decreases by only 0.03%, while consumption drops by 0.08% and output increases by 0.05%.

In other words, compared to the representative firm case, the heterogeneous firm economy exhibits a 15-fold smaller decline in investment, a twofold larger drop in consumption, and a fivefold greater increase in output. Since both models generate similar export responses, these differences reflect contrasting internal adjustment mechanisms. In the representative firm economy, the sharp contraction in investment frees up domestic resources, allowing more home goods to be reallocated toward consumption. This substitution dampens the responses of consumption and output relative to the heterogeneous firm benchmark, where frictions prevent such a sharp reallocation.

4.3 Mechanism

What drives the differences in impulse responses, particularly in aggregate investment, between the representative firm and heterogeneous firm models? One hypothesis is that financial frictions prevent firms from freely adjusting their investment in response to economic shocks. As a result, firms with varying levels of idiosyncratic productivity and cash on hand adopt different investment strategies. From our calibration, the stationary distribution of firms in the benchmark economy with heterogeneous firms is dominated by financially constrained firms. This may explain the muted investment response compared to the representative firm case.

To evaluate this hypothesis, we first examine the investment behavior across firm types. For Type-1 constrained and unconstrained firms, investment is determined by the optimal capital stock $k_t'^{\star}(z)$, which satisfies the following first-order condition:

$$\underbrace{-q_t}_{\text{marginal cost}} + \underbrace{\Lambda_{t+1}\left[q_{t+1}(1-\delta) + \frac{\partial \mathbb{E}_t\left[\text{profit}_{t+1}(z', k')\right]}{\partial k'}\right]}_{\text{marginal revenue}} = 0$$

Table 2: Initial states of chosen firms

Firm Type	Productivity	Capital	Debt
Type-2 Constrained	median(z)	$0.1k'^{\star}$(median(z))	$q\lambda k'^{\star}$(median(z))
Type-1 Constrained	median(z)	$k'^{\star}$(median(z))	$0.5b'^{\star}$(median(z))
Unconstrained	median(z)	$k'^{\star}$(median(z))	$b'^{\star}$(median(z))

$$\implies k_t'^{\star}(z) = \left\{ \frac{\alpha p_{t+1}^{\frac{1}{1-\nu}} \left(\frac{\nu}{w_{t+1}}\right)^{\frac{\nu}{1-\nu}} \mathbb{E}_t \left[z'^{\frac{1}{1-\nu}} \middle| z \right]}{\frac{q_t}{\Lambda_{t+1}} - (1-\delta) q_{t+1}} \right\}^{\frac{1-\nu}{1-(\alpha+\nu)}} \tag{28}$$

In other words, these firms invest up to the point where the marginal cost of capital, given by the current price of capital goods q_t, equals the marginal revenue, defined as the discounted value of the marginal increase in next period's profits and undepreciated capital. When the external shock hits the economy, changes in prices shift both the marginal cost and marginal benefit of investment, thereby altering the optimal capital level $k_t'^{\star}(z)$.

In contrast, a Type-2 constrained firm determines its investment jointly with its borrowing decision, due to a binding collateral constraint. The presence of this constraint introduces a positive Lagrange multiplier, which complicates the firm's optimization problem. The investment decision for a Type-2 constrained firm is characterized by the following condition:

$$k' = \frac{n}{q_t - \lambda Q_t q_{t+1} \Pi_{H,t+1}} \tag{29}$$

Unlike the optimal investment condition for unconstrained or Type-1 constrained firms, which depends on marginal returns and includes the wage rate, the investment decision for Type-2 constrained firms is not directly affected by wages. Instead, investment depends critically on the firm's internal liquidity and financing conditions. Specifically, the investment increases with more cash on hand, as it expands the firm's available resources. It also increases with a higher debt discount rate, reflecting a lower effective interest rate on borrowing.

To illustrate the heterogeneity in investment responses, we select three representative sub-

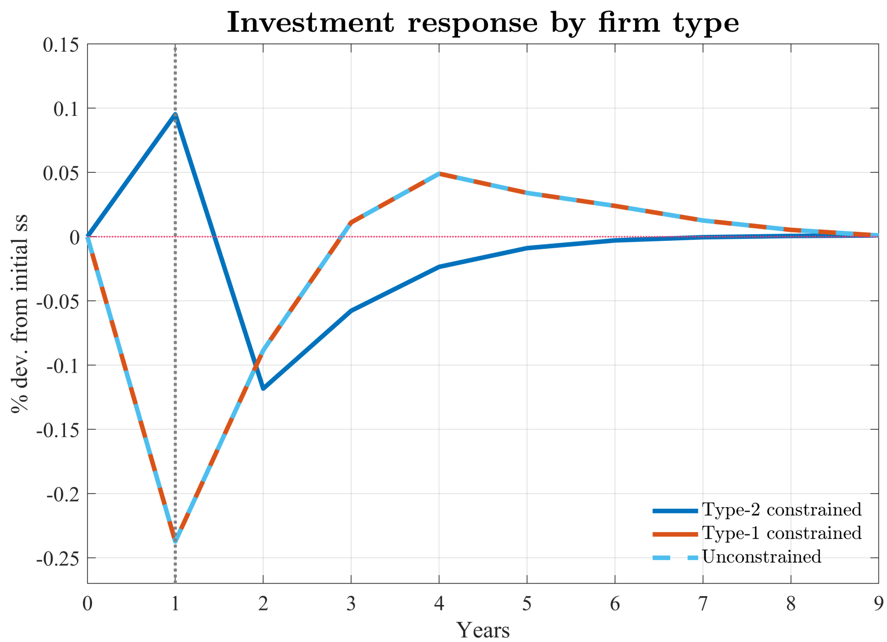

Figure 6: Investment responses of different firm groups

groups of firms from the steady state. All firms in these groups share the same median level of idiosyncratic productivity, but differ in their capital and debt positions. Based on their financial status, the firms are classified as either Type-2 constrained, Type-1 constrained, or unconstrained.

In the Type-2 constrained group, each firm's capital stock is only 10% of its optimal level, and its debt is at the maximum allowed by the collateral constraint. For the Type-1 constrained group, the capital stock is at the optimal level. However, the firm's savings are low, equal to only half of the minimum savings policy. Finally, firms in the unconstrained group hold both the optimal capital stock and the minimum level of precautionary savings, and thus face no binding constraints. We verify the accuracy of these classifications by checking the corresponding policy functions.

Figure 6 shows the investment responses for our three firm groups. After the sudden capital outflow, Type-1 constrained and unconstrained firms cut their investment, just as in the representative firm model,whereas Type-2 constrained firms increase their investments. Because Type-2 firms account for 59.25% of the stationary distribution, their upward invest-

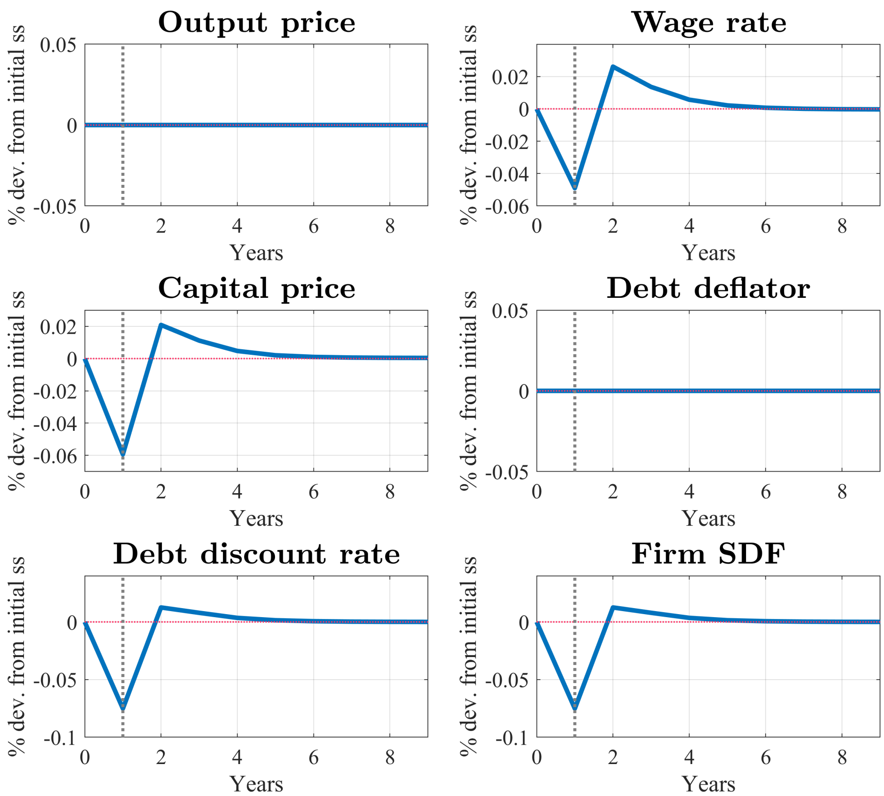

Figure 7: Impulse responses of prices

ment response substantially offsets the investment declines by the other firms. Consequently, aggregate investment falls much less in the first period than in the representative firm economy.

To unpack the drivers of heterogeneous investment responses, we examine the impulse responses of the six key prices that enter firms' optimal investment conditions. Figure 7 shows the transition paths of the intermediate-goods price p, the wage rate w, the capital price q, the debt deflator $1/P_H$, the debt discount rate Q, and the firm's stochastic discount factor Λ.

According to the first-order condition in Equation (28), unconstrained and Type-1 firms' optimal capital choice depends on all of these prices except the debt discount rate. Under our floating exchange rate assumption with a fixed domestic-goods price, both the intermediate-goods price and the debt deflator remain constant, and thus drop out of the investment decision. In period 1, the fall in the price of capital goods lowers the marginal cost of investment, but the marginal revenue, driven by the wage rate and the stochastic discount factor, also declines because wages and the firm's discount factor shifts downwards. In period 2, the price of capital goods rebounds above its steady-state level, mechanically raising marginal revenue, but the simultaneous rise in the wage rate and the drop in the stochastic discount factor more than offset this effect. In sum, the interaction of these price movements leads to a net decrease in the marginal revenue of investment relative to its cost. Consequently, both Type-1 constrained and unconstrained firms reduce their investment in the immediate aftermath of the shock.

Now consider Type-2 constrained firms, whose investment rule is given by Equation (29). In the first period, the fall in the price of capital goods lowers the cost of investment, directly encouraging higher investment. In the second period, the rebound in the price of capital goods not only increases the value of the firm's collateral but also relaxes its binding collateral constraint, since higher capital valuations allow a higher borrowing limit against the same stock of capital. This additional borrowing capacity further amplifies investment. Although the drop in the debt discount rate implies a higher implicit interest cost on new debt, the combined effect of cheaper capital, relaxed collateral requirements, and increased borrowing capacity still yields a net positive impact on the firm's investment.

In summary, different responses from different types of firms contribute to the small aggregate investment response we see in the benchmark economy with heterogeneous firms. Since Type-2 constrained firms have a positive investment response and Type-1 constrained firms and unconstrained firms reduce their investments, the aggregate response will depend crucially on the distribution of these different types of firms in the economy. In contrast, in a representative firm economy, the investment response to a foreign capital reversal is negative and equal to that of an unconstrained firm. Hence, a representative firm model would overestimate the investment response and underestimate the consumption and output responses.

4.4 Comparison to previous literature

In this section, we summarize how our results differ from previous literature, in particular, how our model produces a muted investment response compared to what has been previously found.

Mendoza (2010) develops an open economy business cycle model in which Sudden Stops in foreign capital flows emerge endogenously in certain states of the world, typically following a sustained rise in asset prices and leverage. These crises are triggered when high-leverage conditions cause collateral constraints to bind, leading to a Fisherian debt-deflation spiral: agents are forced to fire-sell capital, which depresses asset prices, tightens borrowing constraints further, and deepens the downturn. In this setting, the tightening of collateral constraints reduces access to external finance, leading to simultaneous declines in consumption, output, and investment. While agents anticipate the possibility of Sudden Stops and accumulate precautionary savings to buffer consumption, the investment response remains sharply contractionary due to the feedback loop between leverage and falling asset prices.

In contrast, our model introduces firm heterogeneity and features a subset of firms that are always financially constrained, even prior to the shock. As a result, the aggregate investment response to a sudden capital flow reversal is substantially dampened relative to Mendoza's findings. Specifically, Type-2 constrained firms, already subject to binding collateral constraints, are less exposed to a rapid deleveraging cycle. When the price of capital falls, their investment actually increases, as the lower capital cost partially offsets their borrowing limitations. Moreover, as capital prices recover in subsequent periods, these firms experience a relaxation of their collateral constraints, further boosting investment. Thus, instead of a generalized debt-deflation spiral, our model highlights a composition effect: the presence and behavior of persistently constrained firms buffers the aggregate investment decline and leads to a delayed but amplified investment response during the recovery phase.

Another closely related study is De Ferra et al. (2020), which also focuses on Hungary around the time of the Global Financial Crisis. While both papers consider a flexible exchange rate regime and a foreign capital withdrawal shock, the mechanisms emphasized differ substantially. De Ferra et al. focus on household heterogeneity. Specifically, they study how the distribution of foreign currency-denominated debt across households with varying marginal propensities to consume (MPCs) shapes the aggregate response. In their framework, the shock leads to a nominal exchange rate depreciation, which revalues foreign-denominated debt and reduces indebted households' wealth. This, in turn, lowers consumption but raises labor supply, leading to higher output. The depreciation also worsens the terms of trade, causing both domestic and foreign consumers to shift from foreign to domestic goods. Investment falls sharply, driven by reduced access to foreign capital and an increase in real interest rates, which depresses capital demand from the representative firm. The amplification mechanism in their model comes from high-MPC households bearing a disproportionate share of foreign currency debt.

In contrast, our paper emphasizes a different transmission channel: firm-level financial frictions in a setting with heterogeneous, financially constrained firms. Rather than focusing on household balance sheets, we examine how the prevalence of collateral constraints among firms shapes aggregate dynamics. The key departure from De Ferra et al. is the introduction of financially constrained firms, particularly Type-2 constrained firms, which fundamentally alters the investment response to the shock. Whereas their model features a representative firm whose investment falls significantly, our heterogeneous firm framework shows that the presence of constrained firms mutes the negative aggregate investment response. This muted investment response, in turn, has important implications for how resources are reallocated and for the dynamics of consumption and output. In short, while De Ferra et al. highlights the composition of household debt as the central amplification mechanism, we show that firm-side heterogeneity and financial frictions can generate distinct and quantitatively significant aggregate effects.

5 Conclusion

This paper embeds firm-level dividend and collateral constraints into an open economy New Keynesian model with heterogeneous firms to examine the propagation of foreign borrowing reversals. A calibrated Sudden Stop shock, a 1% drop in foreign capital inflow followed by mean-reverting recovery, reveals that when a higher share of firms are subject to collateral constraints, aggregate investment volatility is sharply muted compared to a representative firm economy. However, this dampened effect on investment cuts moves resources away from consumption, leaving fewer goods available to households and increasing their labor supply, leading to higher output. These patterns are driven by a composition mechanism: constrained firms, benefiting from higher collateral values, expand investment, and offset cutbacks by other firms.

Our results extend the sudden stop literature by highlighting how cross-firm heterogeneity and binding collateral constraints can dampen investment responses and amplify real side fluctuations. From a policy perspective, the findings suggest that macroprudential regulation of collateral values and borrowing limits can critically shape an open economy's resilience to even modest external funding reversals. An interesting avenue for future research is to study optimal policy design under alternative exchange rate regimes, or in settings with richer models of financial intermediation.

References

Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee, "A corporate balance-sheet approach to currency crises," *Journal of Economic theory*, 2004, *119* (1), 6–30.

Arellano, Cristina, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, June 2008, *98* (3), 690–712.

Auclert, Adrien, Matthew Rognlie, Martin Souchier, and Ludwig Straub, "Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel," Working Paper 28872, National Bureau of Economic Research May 2021.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry, "Really uncertain business cycles," *Econometrica*, 2018, *86* (3), 1031–1065.

Cugat, Gabriela, "Emerging markets, household heterogeneity, and exchange rate policy," 2019 Meeting Papers 526, Society for Economic Dynamics 2019.

Drenik, Andres, Gustavo Pereira, and Diego J. Perez, "Wealth Redistribution after Exchange Rate Devaluations," *AEA Papers and Proceedings*, May 2018, *108*, 552–56.

Feenstra, Robert C, Philip Luck, Maurice Obstfeld, and Katheryn N Russ, "In search of the Armington elasticity," *Review of Economics and Statistics*, 2018, *100* (1), 135–150.

Ferra, Sergio De, Kurt Mitman, and Federica Romei, "Household heterogeneity and the transmission of foreign shocks," *Journal of International Economics*, 2020, *124*, 103303.

Fornaro, Luca, "Financial crises and exchange rate policy," *Journal of International Economics*, 2015, *95* (2), 202–215.

Gopinath, Gita, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez, "Capital allocation and productivity in South Europe," *The Quarterly Journal of Economics*, 2017, *132* (4), 1915–1967.

Guo, Xing, Pablo Ottonello, and Diego J. Perez, "Monetary Policy and Redistribution in Open Economies," *Journal of Political Economy Macroeconomics*, 2023, *1* (1), 191–241.

Iyer, Tara, "Inflation Targeting for India?: The Implications of Limited Asset Market Participation," Working Papers id:7240, eSocialSciences August 2015.

Jo, In Hwan and Tatsuro Senga, "Aggregate consequences of credit subsidy policies: Firm dynamics and misallocation," *Review of Economic Dynamics*, 2019, *32*, 68–93.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante, "Monetary policy according to HANK," *American Economic Review*, 2018, *108* (3), 697–743.

Kátay, Gábor and Zoltán Wolf, "Driving factors of growth in Hungary-a decomposition exercise," Technical Report, MNB Working Papers 2008.

Kehoe, Patrick J. and Ellen R. McGrattan, "Sudden Stops and Output Drops," *American Economic Review*, May 2005, *95* (2), 381–387.

Khan, Aubhik and Julia K Thomas, "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics," *Econometrica*, 2008, *76* (2), 395–436.

_ and _ , "Credit shocks and aggregate fluctuations in an economy with production heterogeneity," *Journal of Political Economy*, 2013, *121* (6), 1055–1107.

_ , Tatsuro Senga, and Julia K Thomas, "Default risk and aggregate fluctuations in an economy with production heterogeneity," *Unpublished Manuscript*, 2020, *5.*

Levinsohn, James and Amil Petrin, "Estimating production functions using inputs to control for unobservables," *The review of economic studies*, 2003, *70* (2), 317–341.

Mendoza, Enrique G, "Sudden stops, financial crises, and leverage," *American Economic Review*, 2010, *100* (5), 1941–1966.

Oskolkov, Aleksei, "Exchange rate policy and heterogeneity in small open economies," *Journal of International Economics*, 2023, *142*, 103750.

Ottonello, Pablo, "Optimal exchange-rate policy under collateral constraints and wage rigidity," *Journal of International Economics*, 2021, *131* (C).

_ and Thomas Winberry, "Financial heterogeneity and the investment channel of monetary policy," *Econometrica*, 2020, *88* (6), 2473–2502.

_ and _ , "Capital, ideas, and the costs of financial frictions," Technical Report, National Bureau of Economic Research 2024.

Salomao, Juliana and Liliana Varela, "Exchange rate exposure and firm dynamics," *The Review of Economic Studies*, 2022, *89* (1), 481–514.

Schmitt-Grohé, Stephanie and Martín Uribe, "Multiple equilibria in open economies with collateral constraints," *The Review of Economic Studies*, 2021, *88* (2), 969–1001.

Sunel, Enes, "Welfare consequences of gradual disinflation in emerging economies," *Journal of Money, Credit and Banking*, 2018, *50* (4), 705–755.

Verner, Emil and Győző Gyöngyösi, "Household Debt Revaluation and the Real Economy: Evidence from a Foreign Currency Debt Crisis," *American Economic Review*, September 2020, *110* (9), 2667–2702.

Villalvazo, Sergio, "Inequality and Asset Prices during Sudden Stops," International Finance Discussion Papers 1388, Board of Governors of the Federal Reserve System (U.S.) March 2024.

Zhou, Haonan, "Open economy, redistribution, and the aggregate impact of external shocks," *Redistribution, and the Aggregate Impact of External Shocks (February 6, 2022)*, 2022.

Appendix

A New-Keynesian Phillips Curve (NKPC)

Recall that each period the retailer i produces differentiated goods using a one-for-one technology $\tilde{y}_{it} = y_{it}$ using the undifferentiated good as input. Retailer i solves the following (nominal) cost minimization problem:

$$\min_{y_{it}} \; P_{H,t} p_t y_{it}$$

$$\text{s.t.} \quad y_{it} \geq \left(\frac{\tilde{p}_{it}}{P_{H,t}}\right)^{-\epsilon} Y_t$$

The inequality constraint comes from the representative final good producer's demand function. Solving it with the Lagrangian:

$$\begin{aligned}
\mathcal{L} &= -P_{H,t} p_t y_{it} + \lambda_{it} \left[y_{it} + (\frac{\tilde{p}_{it}}{P_{H,t}})^{-\epsilon} Y_t \right] \\
\frac{\partial \mathcal{L}}{\partial y_{it}} &= -P_{H,t} p_t + \lambda_{it} = 0
\end{aligned}$$

$$\implies \quad \lambda_{it} = P_{H,t} p_t$$

The Lagrange multiplier λ_{it} can be regarded as the nominal marginal cost. Normalizing it by $P_{H,t}$, we obtain the expression for real marginal cost: $\text{mc}_t := \frac{\lambda_{it}}{P_{H,t}} = p_t$, which is exactly the real relative price of undifferentiated good. Retailer i then maximizes its sum of discounted real profit by setting nominal prices $\{\tilde{p}_{it+s}\}_{s=0}^{\infty}$:

$$\max_{\{\tilde{p}_{it+s}\}_{s=0}^{\infty}} \; \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \frac{u'(C_{t+s})}{u'(C_t)} \frac{P_t}{P_{t+s}} \frac{P_{H,t+s}}{P_{H,t}} \left[\frac{\tilde{p}_{it}}{P_{H,t}} \tilde{y}_{it} - p_t y_{it} - \frac{\varphi}{2} (\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1)^2 Y_t \right]$$

Plugging in the demand function for $\tilde{y}_{it}$ and $\tilde{y}_{it} = y_{it}$, retailer i's problem is equivalent to

$$\max_{\{\tilde{p}_{it+s}\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \frac{u'(C_{t+s})}{u'(C_t)} \frac{P_t}{P_{t+s}} \frac{P_{H,t+s}}{P_{H,t}} \left[\frac{\tilde{p}_{it}}{P_{H,t}} (\frac{\tilde{p}_{it}}{P_{H,t}})^{-\epsilon} Y_t - \text{mc}_t (\frac{\tilde{p}_{it}}{P_{H,t}})^{-\epsilon} Y_t - \frac{\varphi}{2} (\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1)^2 Y_t \right]$$

where $\text{mc}_t = p_t$. The first-order condition with respect to $\tilde{p}_{it}$ is

$$(\epsilon - 1)(\frac{\tilde{p}_{it}}{P_{H,t}})^{-\epsilon} \frac{Y_t}{P_{H,t}} = \epsilon \text{mc}_t (\frac{\tilde{p}_{it}}{P_{H,t}})^{-\epsilon-1} \frac{Y_t}{P_{H,t}} - \varphi \left(\frac{\tilde{p}_{it}}{\tilde{p}_{it-1}} - 1 \right) \frac{Y_t}{\tilde{p}_{it-1}} + \beta\varphi \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \frac{P_{H,t+1}}{P_{H,t}} \left(\frac{\tilde{p}_{it+1}}{\tilde{p}_{it}} - 1 \right) \frac{\tilde{p}_{it+1}}{\tilde{p}_{it}^2} Y_{t+1}$$

In equilibrium all firms behave identically, so $\tilde{p}_{it} = \tilde{p}_t$, and $P_{H,t} = \left(\int_0^1 \tilde{p}_{it}^{1-\epsilon}\, di \right)^{\frac{1}{1-\epsilon}} = \tilde{p}_t$. Substitute the above expressions into the first-order condition, we obtain the nonlinear New-Keynesian Phillips Curve (NKPC):

$$\epsilon - 1 = \epsilon \text{mc}_t - \varphi(\Pi_{H,t} - 1)\Pi_{H,t} + \beta\varphi \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \frac{\Pi_{H,t+1}^2}{\Pi_{t+1}} (\Pi_{H,t+1} - 1) \frac{Y_{t+1}}{Y_t} \right]$$

In steady state, $\Pi_{H,t} = 1$, $\Pi_{H,t+1} = 1$, $\Pi_{t+1} = 1$, $C_{t+1} = C_t = C^{ss}$, $Y_{t+1} = Y_t = Y^{ss}$, $\text{mc}_t = \text{mc}^{ss} = \frac{\epsilon-1}{\epsilon}$. Log-linearize the NKPC around the steady state, we obtain:

$$\begin{aligned}
0 &= \frac{1}{\epsilon \text{mc}^{ss}} \left\{ \epsilon(\text{mc}_t - \text{mc}^{ss}) - \varphi(\Pi_{H,t} - 1) + \varphi\beta(\Pi_{H,t+1} - 1) \right\} \\
0 &= \frac{\epsilon}{\epsilon - 1}(\text{mc}_t - \text{mc}^{ss}) - \frac{\varphi}{\epsilon - 1}(\Pi_{H,t} - 1) + \frac{\varphi\beta}{\epsilon - 1} \mathbb{E}_t(\Pi_{H,t+1} - 1) \\
0 &= \frac{\text{mc}_t - \text{mc}^{ss}}{\text{mc}^{ss}} - \frac{\varphi}{\epsilon - 1}(\frac{\Pi_{H,t} - 1}{1}) + \frac{\varphi\beta}{\epsilon - 1} \mathbb{E}_t(\frac{\Pi_{H,t+1} - 1}{1}) \\
0 &= \hat{\text{mc}}_t - \frac{\varphi}{\epsilon - 1} \hat{\Pi}_{H,t} + \frac{\varphi\beta}{\epsilon - 1} \mathbb{E}_t \hat{\Pi}_{H,t+1} \\
\frac{\varphi}{\epsilon - 1} \hat{\Pi}_{H,t} &= \hat{\text{mc}}_t + \frac{\varphi\beta}{\epsilon - 1} \mathbb{E}_t \hat{\Pi}_{H,t+1} \\
\hat{\Pi}_{H,t} &= \frac{\epsilon - 1}{\varphi} \hat{\text{mc}}_t + \beta \mathbb{E}_t \hat{\Pi}_{H,t+1} \\
\log \Pi_{H,t} &= \frac{\epsilon - 1}{\varphi} \log(\frac{p_t}{p^{ss}}) + \beta \mathbb{E}_t \log \Pi_{H,t+1}
\end{aligned}$$

which is the linearized NKPC (8).

B Current Account Balance

Recall the representative Home household's budget constraint:

$$
\begin{aligned}
&\frac{1}{x_t^{1-\chi}} C_t + \tilde{B}_{t+1}^{HH} + q_t \int k_0 \, d\mu_t^{\text{ent}} \\
&= (1+r_t)\tilde{B}_t^{HH} + w_t L_t + \int F_t(z,k,b) \, d\mu_t + \pi_t^{\text{Retailer}} + \pi_t^{\text{Capital}} + q_t(1-\delta) \int k \, d\mu_t^{\text{exit}} \\
&= (1+r_t)\tilde{B}_t^{HH} + w_t L_t + \int \left[p_t y_t - w_t l + q_t(1-\delta)k - \frac{b}{\Pi_{H,t}} - q_t k' + Q_t b' \right] d\mu_t^{stay} \\
&\quad + \int \left[p_t y_t - w_t l + q_t(1-\delta)k - \frac{b}{\Pi_{H,t}} \right] d\mu_t^{\text{exit}} + \pi_t^{\text{Retailer}} + \pi_t^{\text{Capital}} \\
&= (1+r_t)\tilde{B}_t^{HH} + \left(p_t \int y_t \, d\mu_t + \pi_t^{\text{Retailer}} \right) + \left(w_t L_t - w_t \int l_t \, d\mu_t \right) \\
&\quad - q_t \int [k' - (1-\delta)k] \, d\mu_t^{stay} + q_t(1-\delta) \int k \, d\mu^{exit} + \pi_t^{\text{Capital}} \\
&\quad - \int \frac{b}{\Pi_{H,t}} \, d\mu_t + Q_t \int b' d\mu_t^{stay}
\end{aligned}
$$

In equilibrium the labor market clears, so $w_t L_t - w_t \int l_t \, d\mu_t = 0$. Also, the sum of retailer's dividend and production firm's sale equals the total output: $Y_t = p_t \int y_t \, d\mu_t + \pi_t^{\text{Retailer}}$. The above equation can be rewritten as:

$$
\begin{aligned}
&\frac{1}{x_t^{1-\chi}} C_t + \tilde{B}_{t+1}^{HH} - (1+r_t)\tilde{B}_t^{HH} + \int \frac{b}{\Pi_{H,t}} \, d\mu_t - Q_t \int b' d\mu_t^{stay} \\
&= Y_t - q_t \int [k' - (1-\delta)k] \, d\mu_t^{stay} + q_t(1-\delta) \int k \, d\mu^{exit} - q_t \int k_0 \, d\mu_t^{\text{ent}} + \pi_t^{\text{Capital}} \\
&= Y_t - q_t I_t + q_t I_t - [I_t + \Phi_t] \\
&= Y_t - I_t - \Phi_t
\end{aligned}
$$

Now consider the left-hand side of the equation. From the financial intermediaries' per-

period budget constraint we can derive that

$$
\begin{aligned}
& Y_t - I_t - \Phi_t \\
= & \frac{1}{x_t^{1-\chi}} C_t + \tilde{B}_{t+1}^{HH} - (1+r_t)\tilde{B}_t^{HH} + \int \frac{b}{\Pi_{H,t}} \, d\mu_t - Q_t \int b' d\mu_t^{stay} \\
= & \frac{1}{x_t^{1-\chi}} C_t + (1+r_t)\tilde{B}_t^{\star} - \tilde{B}_{t+1}^{\star} \\
= & C_{H,t} + \frac{1}{x_t} C_{F,t} + (1+r_t)\tilde{B}_t^{\star} - \tilde{B}_{t+1}^{\star}
\end{aligned}
$$

The last equality comes from the consumption expenditure: $\frac{1}{x_t^{1-\chi}} C_t = C_{H,t} + \frac{1}{x_t} C_{F,t}$. Compare the resource constraint of the home economy with the above equation:

$$
\begin{aligned}
Y_t &= C_{H,t} + C_{H,t}^{\star} + I_t + \Phi_t \\
Y_t &= C_{H,t} + \frac{1}{x_t} C_{F,t} + (1+r_t)\tilde{B}_t^{\star} - \tilde{B}_{t+1}^{\star} + I_t + \Phi_t
\end{aligned}
$$

Then, we have the following equation:

$$
C_{H,t}^{\star} = \frac{1}{x_t} C_{F,t} + (1+r_t)\tilde{B}_t^{\star} - \tilde{B}_{t+1}^{\star}
$$

which is equation (24).

C Computation

Our computational method closely follows the approach in Appendix C of Ottonello and Winberry (2024), but we accelerate convergence by avoiding numerical optimization or root-finding when solving firms' policy functions. We summarize the steady-state solution procedure as follows:

Outer loop:

1. Choose a candidate steady-state wage rate w and set a target for aggregate labor supply.

2. **Inner loop:**

   (a) Discretize the idiosyncratic productivity process z.

   (b) Given the wage w, compute the unconstrained capital policy $k^\star(z)$ and the minimum saving policy $b^\star(z)$.

   (c) Construct a three-dimensional grid of firm states (z, k, b).

   (d) For each state (z, k, b), compute cash-on-hand n and the constraint threshold $\bar{n}(z)$. Then, for each pair (n, z):

   - If $n \geq \bar{n}(z)$, the firm is unconstrained:

   $$k' = k^\star(z), \quad b' = b^\star(z).$$

   - If $n < \bar{n}(z)$, the firm faces a financing constraint. Let

   $$\bar{k}(n) = \frac{n}{q - \lambda Q \, \Pi_H \, q}.$$

     Compare $\bar{k}(n)$ to $k^\star(z)$:

– If $\bar{k}(n) \geq k^{\star}(z)$, the firm is Type-1 constrained, choosing

$$k' = k^{\star}(z), \quad b' = \frac{q\, k^{\star}(z) - n}{Q}.$$

– If $\bar{k}(n) < k^{\star}(z)$, the firm is Type-2 constrained, choosing

$$k' = \bar{k}(n), \quad b' = \lambda\, \Pi_H\, q\, \bar{k}(n).$$

(e) Using all firms' policy mappings $(z, k, b) \to (z', k', b')$, build the transition matrix and solve for the stationary distribution $\mu(z, k, b)$.

(f) Compute aggregate labor demand L.

3. If L deviates from the labor-supply target, update w via bisection and repeat the inner loop.

To compute the transition path, we assume the economy reaches the steady state by period $T > 0$. We fix the initial firm distribution at its stationary value and guess the entire price paths $\{x_t, q_t\}_{t=0}^{T}$. Then we iterate:

1. Given $\{x_t, q_t\}$, solve for all other prices using market-clearing and debt-pricing conditions.

2. At each t, compute firms' policy functions as above.

3. Aggregate demand and supply for $\{C_t, I_t\}_{t=0}^{T}$; if they do not coincide, update $\{x_t, q_t\}$ and repeat.

In our implementation, we solve for the equilibrium paths $\{x_t, q_t\}$ using MATLAB's `fsolve` routine.